Name:_____

SM3 7.4 Log Properties

We can manipulate log expressions, in order to simplify them or make equations that contain them easier to solve. The first rule, $\log_b xy = \log_b x + \log_b y$ is proven below:

$b^f = x \longrightarrow f = \log_b x$ $b^g = y \longrightarrow g = \log_b y$	We arbitrarily picked f and g to be the number of times b needed to be multiplied to be x and y.
$xy = b^f b^g$	Multiplication
$xy = b^{f+g}$	Exponential Rule
$\log_b xy = \log_b b^{f+g}$	$\log_b$ both sides
$\log_b xy = f + g$	Evaluate $\log_b$ on right side
$\log_b xy = \log_b x + \log_b y$	Substitution for f and g

The proof of the second rule, $\log_b \frac{x}{y} = \log_b x - \log_b y$, is like the first rule but with a negative value for g.

The proof of the third rule, $\log_b x^p = p \log_b x$, is like the first but with several copies of f being used.

The conversion of base rule, $\log_b x = \frac{\log x}{\log b}$, allows you to move from $\log_b$ to any $\log$ that you like. We used base 10 for convenience, since most calculators operate easily in base 10.

Example: Expand the following using logarithmic properties.

$\log_3 5x$

$\log_3 5 + \log_3 x$

Example: Expand the following using logarithmic properties.

$\log \sqrt[3]{x^2 + 3x + 2}$

$\frac{1}{3}\log(x^2 + 3x + 2)$

$\frac{1}{3}\log(x + 2)(x + 1)$

$\frac{1}{3}\log(x + 2) + \frac{1}{3}\log(x + 1)$

Example: Rewrite the following in base 10.

$\log_6 7$

$\frac{\log 7}{\log 6}$

HW7.4

Expand the following using logarithmic properties. Simplify if appropriate.

1. $\log[(2)(3)]$ 2. $\log\frac{2}{3}$ 3. $\log 2^3$ 4. $\log_4 3^x$

5. $\log 2x$ 6. $\log_2\frac{x}{2}$ 7. $\log_5 x^3$ 8. $\log_4[x(2-x)]$

9. $\log_3\frac{5}{x+3}$ 10. $\log_2(x+1)^4$ 11. $\log\sqrt{x}$ 12. $\log\sqrt[3]{x+3}$

13. $\log_4[x(x-2)^4]$ 14. $\log_3[x(x+1)]^5$ 15. $\log\sqrt[4]{\frac{x+1}{x-1}}$ 16. $\log_3\frac{\sqrt{x}}{3+x^2}$

17. $\ln(2x)$ 18. $\ln\frac{x}{3}$ 19. $\ln x^3$ 20. $\ln[x(x-1)^4]$

21. $\ln[(x-2)(x+3)]^2$ 22. $\ln(x^2-16)$ 23. $\ln\frac{x^2+3x-4}{x^2-25}$

24. $\ln[3x^4(9-2x)]$ 25.. $\ln\frac{x^2-4}{x+2}$ 26. $\ln(x^4+3x^3+2x-1)^2$

Rewrite the following within one logarithm.

27. $\log_2 x+\log_2 y$ 28. $3\log x$ 29. $\log_2 x-\log_2 y$

30. $\frac{1}{2}\log_3(x-3)$ 31. $2\log_5 x+\log_5(4+x)$ 32. $\log(x-2)-\log(x+2)$

33. $3\log_2(x^2)+3\log_2(x+10)$ 34. $\log x+\log(x-2)+\log(x^3+3)$

35. $\log_4(x+3)+\log_4(x-3)-\log_4(x+7)$

36. $\frac{2}{3}\log_7 x-\frac{1}{3}\log_7(4-x)$

37. $\ln x+\ln 2$ 38. $\ln x-\ln 2$ 39. $4\ln x$ 40. $\ln(x+1)+\ln 2$

41. $3\ln x+2\ln(x+3)$ 42. $\ln(x-5)+\ln(x+5)$ 43. $\frac{1}{2}\ln x-\ln(4-x)$

44. $\ln e^0+e^{\ln x}$ 45. $\ln x+\ln y-\ln z$ 46. $3[\ln(x-2)+\ln(3+x)]$

Rewrite the following in base 10.

47. $\log_2 3$ 48. $\log_8 4$ 49. $\log_3 x^2$ 50. $\log_3(x-2)$

51. $\log_7(x+5)$ 52. $\log_3\sqrt[3]{x-4}$ 53. $\log_5 12x$ 54. $\log_4(x+7)$

Rewrite the following logarithms as Natural Logarithms.

55. $\log_4 10$ 56. $\log_2 4$ 57. $\log_4 x^3$ 58. $\log_7(x+10)$

59. $\log_3(x-5)$ 60. $\log_3\sqrt[4]{2x-4}$ 61. $\log_2 3x$ 62. $\log_5(3x+7y)$